Note $c = 3 \times 10^8 \text{ m/s}$, $\hbar = 1.055 \times 10^{-34} \text{ Js}$, $m_0 = 9.11 \times 10^{-31} \text{ kg}$, $q = 1.6 \times 10^{-19} \text{ C}$, $\varepsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$, $k = 1.38 \times 10^{-23} \text{ J/K}$.

30 pts.

1. Calculate the commutator $[p_x, x]$ and explain what the outcome implies for the simultaneous measurement of $p_x$ and $x$.

30 pts.

2. In an infinitely deep, 1D, well quantum effects become apparent when the ground state energy level is comparable to the thermal energy, $kT/2$, of an electron for one direction. Calculate the well width in Angstrom where this criterion is met in GaAs where $m_e = 0.08 m_0$ and $T = 300 \text{ K}$.

40 pts.

3. Consider a finite height conduction band quantum well as it forms in the Al$_x$Ga$_{1-x}$As/GaAs system.

3a. Sketch the energy profile perpendicular to the interface and indicate the first three energy states and their associated wave functions.

3b. Assuming that the Fermi-level $E_F$ lies between $E_2$ and $E_3$ and that all allowed states below $E_F$ are filled and above $E_F$ are empty, derive a simple algebraic expression for the electron density in the well as a function of $E_F$, $E_1$ and $E_2$ using the 2D density of states concept.

$$n = \frac{m}{\hbar^2} \int_{E_1}^{E_F} \frac{N(E) \, dE}{2E_F - E_1 - E_2}$$
1. See book pg 41.

Non-commuting variables cannot be measured with infinite accuracy simultaneously. They are subject to the Heisenberg Uncertainty Principle.

\[ \frac{\hbar T}{2} = \frac{1.38 \times 10^{-23} \times 300}{2} = 2.07 \times 10^{-21} \text{J} \]

\[ E_n = \frac{\hbar^2 k^2}{8m a^2} \]

\[ \sqrt{2a} \]

\[ a^2 = \frac{\hbar^2 (1.055 \times 10^{-34})^2}{8 	imes 0.08 \times 9.11 \times 10^{-31} \times 2.07 \times 10^{-21}} = 9.102 \times 10^{-17} \text{m}^2 \]

\[ a = 9.54 \times 10^{-9} \text{m} = 95.4 \text{Å} \]

Well width = 2a = 190.8 Å